the trend curve of best fit.

Remark. Change of Origin. Usually, the values of t are for different years, say, 1990, 1991,..., 1999 and thus computation of $\sum t$, $\sum t^2$, $\sum t^3$, $\sum t^4$, etc., and hence the solution of equations (11·12) and (11·13) for linear trend or equations (11·15) for parabolic trend is quite tedious and time consuming. However, it may be remarked that the time variable t in the time series has no magnitudinal value but it has only positional or locational importance. Hence, we can shift the origin in the time variable according to our convenience and assign it the consecutive values 0, 1, 2,..., etc.. The time period allotted to the value 0 is known as the *period of origin*. This might slightly facilitate the solution of the normal equations. However, the algebraic computations can be simplified to a great extent by shifting the origin in time variable t to a new variable x in such a way that we always get $\sum x = \sum x^3 = 0$. The *technique* is explained below and *can be applied only if the values of t are given to be equidistant,* say, at an interval h.

If n, *the number of time series values is odd, then* the transformation is :

$$x = \frac{t - \text{middle value}}{\text{Interval } (h)} \qquad \text{...(11·16)}$$

Thus, if we are given yearly figures for, say, 1990, 1991, 1992, ..., 1996, *i.e.*, $n = 7$, then

$$x = \frac{t - \text{middle year}}{1} = t - 1993 \qquad \text{...(*)}$$

Putting t = 1990, 1991, 1992, ..., 1996 in (*), we get $x = -3, -2, -1, 0, 1, 2$ and 3 respectively so that $\sum x = \sum x^3 = 0$.

If n is even then, the transformation is :

$$x = \frac{t - (\text{Arithmetic mean of two middle values})}{\frac{1}{2}(\text{Interval})} \qquad \text{...(11·17)}$$

Thus, if we are given the yearly values for, say, 1995, 1996, 1997,..., 2002, then

$$x = \frac{t - \frac{1}{2}(1998 + 1999)}{\frac{1}{2}} = 2\,(t - 1998{\cdot}5) = 2t - 3997 \qquad \text{...(**)}$$

Putting $t = 1995, 1996, \ldots, 2002$ in (**), we get respectively :

$$x = -7, -5, -3, -1, 1, 3, 5, 7 \qquad \text{so that} \qquad \sum x = \sum x^3 = 0$$

The transformations (*) or (**) will always give $\sum x = 0 = \sum x^3$, and this reduces the algebraic calculations for the solution of normal equations to a great extent. For example, for the linear trend

$$y = a + bx, \qquad \ldots(11\cdot18)$$

where x is defined either by (11·16) or (11·17) according as n is odd or even, the normal equations for estimating a and b become :

$$\sum y = na + b\sum x \qquad \text{and} \qquad \sum xy = a\sum x + b\sum x^2$$

but $\sum x = 0$. Hence these equations give :

$$\sum y = na \quad \text{and} \quad \sum xy = b\sum x^2 \quad \Rightarrow \quad a = \frac{\sum y}{n} \quad \text{and} \quad b = \frac{\sum xy}{\sum x^2} \qquad \ldots(11\cdot19)$$

With these values of a and b, (11·18) gives the equation of the trend line.

Similarly, for the parabolic trend : $y = a + bx + cx^2$, ...(11·20)

the normal equations for estimating a, b and c are

$$\left.\begin{aligned} \sum y &= na + b\sum x + c\sum x^2 \\ \sum xy &= a\sum x + b\sum x^2 + c\sum x^3 \\ \sum x^2y &= a\sum x^2 + b\sum x^3 + c\sum x^4 \end{aligned}\right\} \text{ which reduce to } \left.\begin{aligned} \sum y &= na + c\sum x^2 \\ \sum xy &= b\sum x^2 \\ \sum x^2y &= a\sum x^2 + c\sum x^4 \end{aligned}\right\} \begin{aligned} &\ldots(i) \\ &\ldots(ii) \quad [\because \sum x = \sum x^3 = 0] \\ &\ldots(iii) \end{aligned}$$

Equation (ii) gives the value of $b = \dfrac{\sum xy}{\sum x^2}$ and equations (i) and (iii) can be solved simultaneously for a and c. With these values of a, b and c the curve (11·20) becomes the parabolic trend curve of best fit.

Fitting of Exponential Trend. Th...